

THE FIRST MEXICO/USA LOGIC FEST

1 Schedule (January 10-13, 2018)

1.1 Wednesday 10

- 09:00-09:30 Welcome
- 09:30-10:20 Arthur W. Apter, *Strong compactness, Easton functions and indestructibility*
- 10:30-11:00 Coffee break
- 11:00-11:50 Rodrigo Jesús Hernández Gutiérrez, *T-algebras and magic G_δ 's*
- 12:00-12:50 David Fernández Duque, *The Caristi fixed point theorem, the Ekeland variational principle, and strong systems of arithmetic*
- 13:00-14:30 Lunch
- 14:30-15:20 Víctor Torres Pérez, *Conjectures of Rado and Chang, Square principles, Tree Properties and Bounded Forcing Axioms*
- 15:30-16:00 Coffee Break
- 16:00-16:40 Jouko Väänänen, *Internal categoricity*

1.2 Thursday 11

- 09:30-10:20 Joan Bagaria, *The role of large cardinals in the semantics of Generalized Provability Logics*
- 10:30-11:00 Coffee break
- 11:00-11:50 Christian Rosendal, *TBA*
- 12:00-12:50 Mirna Džamonja, *Crisis in Foundations: is it really happening?*
- 13:00-14:30 Lunch
- 14:30-15:20 Ulises Ariet Ramos García, *Extremally disconnected topological groups*
- 15:30-16:10 Andrés Villaveces, *Some interactions between model theory and set theory*
- 17:00-19:00 Cocktail reception

1.3 Friday 12

- 09:30-10:20 Anush Tserunyan, *Hyperfinite ergodic subgraphs*
- 10:30-11:00 Coffee break
- 11:00-11:50 David Asperó, *Few new reals*
- 12:00-12:50 Dima Sinapova, *TBA*
- 13:00-14:30 Lunch
- 14:30-15:20 John R. Steel, *Mouse pairs. Part I*
- 15:30-16:00 Coffee Break
- 16:00-16:40 John R. Steel, *Mouse pairs. Part II*
- 16:50-17:30 Juan Pablo Aguilera Ozuna, *TBA*
- 18:30-20:30 Social Dinner

1.4 Saturday 13

- 09:00-09:40 John Krueger, *Club Isomorphisms on Higher Aronszajn Trees*
- 09:50-10:30 Michael Hrusak, *TBA*

2 List of abstracts

Title: TBA

Speaker: Juan Pablo Aguilera Ozuna, Vienna University of Technology

Title: Strong Compactness, Easton Functions, and Indestructibility

Speaker: Arthur W. Apter, CUNY (Baruch College and the Graduate Center)

Abstract: I will discuss realizing Easton functions in the presence of non-supercompact strongly compact cardinals and connections with indestructibility.

Title: Few new reals

Speaker: David Asperó, School of Mathematics, University of East Anglia

Abstract: The method of iterated forcing with symmetric systems of models as side conditions can be successfully applied to the construction of models of set theory with continuum large. In this talk I will describe a variant of this method giving rise to models of CH. One interesting feature of the resulting forcings is that they add new reals, although only \aleph_1 -many of them. I will focus on one specific application of this method. This is joint work with Miguel Angel Mota.

Title: The role of large cardinals in the semantics of Generalized Provability Logics

Speaker: Joan Bagaria, University of Barcelona and ICREA

Abstract: The logic GLP, namely the the polymodal extension of the classical Gödel-Löb provability logic GL to infinitely-many modal operators $[n]$, completely axiomatizes the arithmetical properties of the n -provability operators (i.e., provability in Peano's Arithmetic together with all true Π_n truths). GLP has a natural topological semantics given by polytopological ordinal spaces. The completeness of GLP under this semantics is a long-standing open problem. We shall present some recent results showing that the problem necessarily involves the existence of large cardinals equivalent to strong forms of reflection of stationary sets. We shall also discuss some recent work on the consistency strength of these cardinals.

Title: Crisis in Foundations: is it really happening?

Speaker: Mirna Džamonja, School of Mathematics, University of East Anglia and associated Member at IHPST, Université Panthéon-Sorbonne

Abstract: We discuss the connections and differences between the ZFC set theory and univalent foundations and answer the above question in the negative.

Title: The Caristi fixed point theorem, the Ekeland variational principle, and strong systems of arithmetic

Speaker: David Fernández-Duque, Ghent University

Abstract: A *Caristi system* is a triple (X, T, f) , where X is a complete metric space, $f: X \rightarrow (0, \infty)$ is a lower semi-continuous function, and $T: X \rightarrow X$ is an arbitrary transformation such that, for all $x \in X$,

$$d(x, T(x)) \leq f(x) - f(T(x)).$$

Caristi's fixed point theorem (CFP) states that any Caristi system has a fixed point; that is, there is $x_* \in X$ such that $T(x_*) = x_*$. It was proven by Caristi using transfinite sequences of the form $(x_\xi)_{\xi < \omega_1} \subseteq X$, defined recursively so that $x_{\xi+1} = T(x_\xi)$ for all ξ and the sequence converges at limit ordinals.

Caristi's theorem can also be derived from *Ekeland's variational principle (EVP)*, which states that if $f: X \rightarrow (0, \infty]$ is lower semi-continuous, then for every $\varepsilon > 0$, f has an ε -critical point; that is, a point $x_* \in X$ such that whenever

$$\varepsilon d(x_*, x) \leq f(x_*) - f(x),$$

it follows that $x = x_*$. Such an x_* is not necessarily a local minimum for f , but via a small perturbation one can obtain a function \tilde{f} which attains its minimum value at x_* .

We analyze these two results in the context of reverse mathematics, where metric spaces are assumed separable and coded in the standard way. Among the results obtained, we have that, over RCA_0 :

- When restricted to the case where f is continuous and X is compact, CFP and EVP are both equivalent to WKL_0 .
- When restricted either to the case where f is continuous or to the case where X is compact, CFP and EVP are both equivalent to ACA_0 .
- CFP for Baire or Borel T is equivalent to TLPP_0 , the Σ_α -relative leftmost path principle for every well-ordering α .
- The unrestricted EVP is equivalent to $\Pi_1^1\text{-CA}_0$.

These theories are all defined over the language of second-order arithmetic and we mention them in strictly increasing order of strength.

In this talk we will present the original proofs of these results, which *prima facie* require principles stronger than those mentioned. We will then discuss how one can avoid such principles using suitable approximation techniques, and show via reverse mathematics that the above-mentioned theories are optimal. This is joint work with Paul Shafer, Keita Yokoyama, and Henry Towsner.

Title: T -algebras and magic G_δ 's

Speaker: Rodrigo Jesús Hernández Gutiérrez, Universidad Autónoma Metropolitana

Abstract: In 2013, Alan Dow and Saharon Shelah constructed a Efimov space under the assumption $\mathfrak{d} = \mathfrak{c}$. The two main ingredients of their proof were a relatively new method to construct topological spaces introduced by Koszmider and a special trick that preserves sets of type G_δ during the construction. The technique introduced by Koszmider consists in taking the Stone space of what he calls T -algebras, a special type of Boolean algebras constructed using a subtree of $2^{<\mathfrak{c}}$. Jointly with Angelo Bella and Alan Dow, we have obtained another application of this method in the construction of a compact, sequentially compact space of uncountable tightness that does not contain topological copies of ω_1 . In this talk the speaker would like to introduce these notions and try to give a sketch of the proof of his joint result.

Title: TBA

Speaker: Michael Hrusak, Centro de Ciencias Matemáticas, UNAM

Title: Club Isomorphisms on Higher Aronszajn Trees

Speaker: John Krueger, University of North Texas

Title: Extremally disconnected topological groups

Speaker: Ulises Ariet Ramos García, Centro de Ciencias Matemáticas, UNAM¹

Abstract: A topological space is called *extremally disconnected* (or *ED* for short) if the closure of every open set is open. In 1967, Arhangel'skii posed the problem of the existence in ZFC of a nondiscrete Hausdorff ED topological group. Recently Reznichenko and Sipacheva proved that the nonexistence of a countable nondiscrete Hausdorff ED group is consistent with ZFC. The general case is still open and, in fact, the uncountable version of Arhangel'skii's problem remains largely unexplored.

In this talk we will present some results about countable and uncountable ED groups. In particular, we will construct, using \diamond , a nondiscrete Hausdorff extremally disconnected topological group of size ω_1 where every countable subset is close and discrete. This talk is based on joint work with Michael Hrušák and Carlos Azarel Martínez-Ranero.

Title: TBA

Speaker: Christian Rosendal, University of Illinois at Chicago

Title: TBA

Speaker: Dima Sinapova, University of Illinois at Chicago

¹Partially supported by the PAPIIT grant IA100517.

Title: Mouse pairs

Speaker: John R. Steel, University of California, Berkeley

Abstract: Many of the basic results of inner model theory are best formulated as theorems about pairs (P, Σ) such that P is a premouse of some kind, and Σ is an iteration strategy for P that *normalizes well* and has *strong hull condensation*. We call such pairs *mouse pairs*.

After defining “mouse pair” more carefully, we shall outline a proof of the Comparison Theorem for such pairs. Then we shall show how one variety of mouse pair, the *least branch hod pairs*, can be used to analyze the hereditarily ordinal definable sets in models of the Axiom of Determinacy.

Title: Conjectures of Rado and Chang, Square principles, Tree Properties and Bounded Forcing Axioms

Speaker: Víctor Torres Pérez, Vienna University of Technology

Abstract: Rado’s Conjecture (RC) in the formulation of Todorćević is the statement that every tree T that is not decomposable into countably many antichains contains a subtree of cardinality \aleph_1 with the same property. Todorćević has shown the consistency of this statement relative to the consistency of the existence of a strongly compact cardinal.

Todorćević also showed that RC implies the Singular Cardinal Hypothesis, a strong form of Chang’s Conjecture, the continuum is at most \aleph_2 , the negation of $\square(\theta)$ for every regular $\theta \geq \omega_2$, etc. These implications are very similar to the ones obtained from traditional forcing axioms such as MM or PFA. However, RC is incompatible even with $\text{MA}(\aleph_1)$.

In this talk we will give an overview of our results with different coauthors obtained in the last few years together with recent ones, involving RC, certain weak square principles and instances of tree properties, together with certain bounded forcing axioms. These new implications seem to continue suggesting that RC is a good alternative to forcing axioms. We will discuss to which extent this may hold true and where we can find some limitations.

We will end the talk with some open problems and possible new directions.

Title: Hyperfinite ergodic subgraphs

Speaker: Anush Tserunyan, University of Illinois at Urbana Champaign

Abstract: Using new methods, we prove that any locally countable ergodic Borel graph on a standard probability space admits an ergodic hyperfinite subgraph. We will discuss the main result and some ingredients of the proof.

Title: Internal categoricity

Speaker: Jouko Väänänen, University of Helsinki

Abstract: Informally speaking, the categoricity of an axiom system means that its non-logical symbols have only one possible interpretation that renders the axioms true. Although non-categoricity has become ubiquitous in the second half of the 20th century whether one looks at number theory, geometry or analysis, the first axiomatizations of such mathematical theories by Dedekind, Hilbert, Huntington, Peano and Veblen were indeed categorical. A common resolution of the difference between the earlier categorical axiomatizations and the more modern non-categorical axiomatizations is that the latter derive their non-categoricity from Skolem's Paradox and Gödel's Incompleteness Theorems, while the former, being second order, suffer from a heavy reliance on metatheory, where the Skolem-Gödel phenomenon re-emerges. Using second order metatheory to avoid non-categoricity of the meta-theory would only seem to lead to an infinite regress. In this talk we maintain that internal categoricity breaks this traditional picture. It applies to both first and second order axiomatizations, although in the first order case we have so far only examples. It does not depend on the meta-theory in a way that would lead to an infinite regress. And it covers the classical categoricity results of early researchers. In the first order case it is weaker than categoricity itself, and in the second order case stronger. We give arguments suggesting that internal categoricity is the "right" concept of categoricity.

Title: Some interactions between model theory and set theory

Speaker: Andrés Villaveces, Universidad Nacional de Colombia

Abstract: I will describe some interactions between set theory and model theory: the role of reflection principles and large cardinals in some central issues in model theory, with recent and also more classical perspectives.